Angular Acceleration and Moment of Inertia

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Abstract:

In this experiment, the moment of inertia of a large plate was experimentally determined and then compared with the theoretical moment of inertia in accordance with equation I = (½)mR2. To experimentally determine the moment of inertia of the plate, several trials took place in which a known torque was applied to it and its resulting angular acceleration was measured. By graphing the resulting angular acceleration against the mass of the hanging object that resulted in the torque, an average was found, which was then used with standard rotational motion equations to determine the average resistance to that motion, I. The experiment determined that the theoretical determination of I was accurate, as the experimental value had little difference from the theoretical value.

Introduction:

What we’re finding:

We will be varying the magnitude of the force applied to a disk at a set radius to see how it affects angular acceleration. These measurements will then be used to find the moment of inertia of the rotating body, which can be compared to the theoretical moment of inertia.

Set up:

The set up consists of a large circular plate free to rotate about a low friction vertical axis through the use a thread wound around a small pulley attached to the plate with a hanging weight attached to it. The pulley has a motion sensor to collect data. The pulley has a set radius, and the magnitude of the force can be controlled by varying the mass of the hanging body.

ADD FIGURE HERE

Equations:

For the torque applied from the tension of the string

𝜏 = 𝑇­­­­­­­­­1r = 𝐼M𝛼1

T1 = 𝐼M𝛼1/r

For the tensions on both sides of the pulley due to no slippage with the string in the pulley

𝜏2 − 𝜏1 = 𝑇2𝑅 − 𝑇1𝑅 = 𝐼𝑝𝛼2

In which 𝑇2 can be determined to be negligible

Using Newton’s 2nd law

𝑚𝑔 − 𝑇2 = 𝑚𝑎2

𝑇2 = 𝑚𝑔 – 𝑚𝑎2

Substitution gives

𝑅 (𝑚𝑔 – 𝑚𝑎2 – (𝐼M𝛼1/r)) = 𝐼𝑝𝛼2

Which is solved for 𝛼2 to give

𝛼2 = 𝑔/(𝐼𝑝/ 𝑚𝑅+ 𝐼𝑀𝑅 /𝑚𝑟2 + R))

Which also gives

EQ 1: 𝛼2 = mR 𝑔/Im

When I­p is assumed to be negligible

A frictional torque is also present which opposes the rotational motion

EQ 2: 𝜏𝑇𝑂𝑇𝐴𝐿 = 𝜏 – 𝜏­fric = 𝑇­𝑟 – 𝜏­fric = 𝐼𝑀𝛼1

In applying the slope determined later in the procedure to the previous equations, perform a substitution for Im using

EQ 3: Im = gr/slope

Equipment:

* PASCO Rotational Dynamics apparatus
* PASCO Rotary Motion Sensor
* Laptop computer running Logger Pro software
* LabQuest Mini interface
* Vernier calipers
* Pan/beam type balance
* Meter stick
* Threaded weight set (or hex nuts)

Procedure:

* Measure the following:
  + Mass of the large plate with a triple beam balance
  + Diameter of the large plate with a ruler
  + Diameter of the largest step pulley with a Vernier caliper
  + Diameter of largest RMS pulley with a Vernier caliper
  + The mass of the object to be hung to provide the tension – in this case a measurement of the mass of a core with subsequent measurements of mass as nuts were attached – measure with a triple beam balance
* Assemble the setup and open logger pro to collect RMS pulley data
* Determine a correction factor for the rotation of the RMS pulley such that the sensor detects the angular rotation of the large plate instead of the RMS pulley
  + This was achieved through the use of a built in correction factor in the logger pro program, which measured the rotation of the RMS pulley across one full revolution of the large plate
* Collect angular velocity vs time data and compare with the torque provided to the large plate
  + Wind the string about the large plate such that the hanging max is a few centimeters from the RMS pulley
  + Start data collection on logger pro and then release the mass, allowing it to fall and rotate the large plate
  + Perform 6 total trials, varying the mass hanging
  + Create a plot of the angular acceleration from the six trials as a result of the mass
  + Add a linear fit and determine the slope

Analysis:

* Determine Im using EQ1, utilizing EQ3
* Determine 𝜏fric from EQ2, utilizing EQ3
* Propagate uncertainty throughout all calculations to determine the uncertainty in I­­m

Conclusion:

For this experiment, the experimental data largely supports the theoretical calculations. The estimated moment of inertia had a value of 7.980 \* 10-3 kg-m, and the experimentally calculated moment of inertia had a value of 8.421 \* 10-3 kg-m. This slight difference makes sense, as unaccounted for frictional forces and the moment of inertia of the RMS pulley would result in the large plate showing additional resistance to motion, making for a calculation of a larger than actual moment of inertia.